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Centralized Ensemble-Based Trajectory Planning of Cooperating Sensors for Estimating Atmospheric Dispersion Processes ^{*}

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Abstract. Optimal coordination of multiple sensors is crucial for efficient atmospheric dispersion estimation. The proposed approach adaptively provides optimized trajectories with respect to sensor cooperation and uncertainty reduction of the process estimate. To avoid the time-consuming solution of a complex optimal control problem, estimation and vehicle control are considered separate problems linked in a sequential procedure. Based on a partial differential equation model, the Ensemble Transform Kalman Filter is applied for data assimilation and generation of observation targets offering maximum information gain. A centralized model-predictive vehicle controller simultaneously provides optimal target allocation and collision-free path planning. Extending previous work, continuous measuring is assumed, which attaches more significance to the course of the trajectories. Local attraction points are introduced to draw the sensors to regions of high uncertainty. Moreover, improved target updates increase the sampling efficiency. A simulated test case illustrates the approach in comparison to non-attracted trajectories.

Keywords: Adaptive Observation, ETKF, State Estimation, Mobile Sensors, Cooperative Control

1 Introduction

Depending on the weather conditions, the dispersion of gaseous material in the atmosphere easily turns into a large-scale highly dynamic process. In order to understand its characteristics and predict future impacts, fast and accurate state estimation is required. The use of robotic systems for autonomous data gathering has been increasingly considered in this context [8]. Sensor-equipped autonomous

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vehicles are able to adapt their movement to a changing environment, which is particularly beneficial when dealing with atmospheric dispersion. Employing multiple mobile sensors permits to cover larger domains and optimal cooperation among them increases the efficiency of the estimation procedure.

As the dispersion dimensions, in general, prohibit pattern-based sampling or global exploration, immediate processing of the gathered data and adaptive sensor motion planning is essential. Information obtained from the sensors and the predictions of an underlying process model can be combined by a data assimilation method to estimate the current process state. In this way, uncertainties stemming from observation noise and model errors are reduced. Using a partial differential equation (PDE) model, more accurate forecasts can be obtained since the physics and the dynamic behavior of the dispersion process are considered.

Related approaches often avoid detailed PDE models and instead use simple models, such as Gaussian processes [15] or qualitative models [7]. They can provide results in a very short time and are frequently used in distributed systems. However, as important characteristics of the process dynamics are not considered, only inaccurate approximations can be obtained.

Adaptive observation strategies based on PDE models are commonly used in large-scale systems, e.g. for numerical weather prediction. Examples are the singular vector technique [3], the gradient method [5] or the Ensemble Transform Kalman Filter (ETKF) [2, 12]. Due to the huge system dimensions, though, vehicle dynamics are not considered in these applications.

Only few publications focus on adaptive observation strategies combining PDE models and vehicle dynamics. While [14] and [16] deal with parameter estimation, [11] and [17] consider state estimation problems in conjunction with data assimilation. All these approaches involve a sophisticated optimal control problem subject to the process model, the covariance evolution, and the vehicle dynamics. Solving such complex problems is hard and time-consuming, especially regarding the real-time requirements of the application.

This is why [4] and [6] utilize computationally efficient suboptimal sensor guidance schemes based on the gradient of the mutual information rate and Lyapunov stability arguments, respectively. Although multiple sensor vehicles are employed, the cooperation among them is either not explicitly addressed or cannot be considered optimal.

In previous work [13], a centralized adaptive observation strategy was proposed that combines PDE-based process models and optimally cooperating mobile sensors for online state estimation. Instead of solving a sophisticated optimal control problem, state estimation and vehicle control are considered separate problems that are linked in a repeating sequential procedure: The process estimate, based on a PDE model, is improved by assimilation of new sensor data using an ETKF approach. Based on the estimate's error covariance matrix, measurement locations providing maximum information gain are determined and are used by a model-predictive controller to guide the sensor vehicles based on a discrete-continuous linear optimization program. This results in a significant gain of computational efficiency compared to solving a (nonlinear) optimal con-

trol problem incorporating process and vehicle dynamics. The focus was set on application scenarios where measurements are expensive so that they are only performed if one of the specified locations is reached by a sensor vehicle.

In contrast, this work adapts the strategy presented in [13] to exploit the advantages of a sensor system able to measure at every time step. Now that information is gathered en route, more importance is attached to the course of the trajectories leading to the target locations. Introducing additional local attraction points draws the sensor vehicles to regions afflicted with high uncertainty. Furthermore, a revised handling of target points accounts for the permanently changing error covariance.

The rest of the paper is organized as follows. Sections 2 and 3 give a short overview of the basic methodologies employed for state estimation and vehicle control, respectively. In Section 4, the adaptive motion planning approach is summarized. Testcases and evaluation results are presented in Section 5 followed by concluding remarks in Section 6.

2 Model and State Estimation

2.1 Process Model

The aim of the proposed approach is to estimate the state of a dynamic transport process that can be described by a PDE of the form

$$\frac{\partial \chi}{\partial t} = f(\chi(t), \nabla \chi(t), \Delta \chi(t), \mathbf{w}(t), \nabla \mathbf{w}(t)), \quad (1)$$

with the dispersing entity χ to be estimated and the underlying velocity field \mathbf{w} . Applying a spatial and temporal discretization scheme, an approximate solution of the PDE can be found by solving an equation of the form

$$\boldsymbol{\chi}_{i+1}^f = M_i[\boldsymbol{\chi}_i^f] \quad (2)$$

where M_i is the model operator obtained by the discretization scheme and $\boldsymbol{\chi}$ is the state vector, which contains values of χ at certain, discrete spatial positions. With this formulation, the state vector at time t_{i+1} can be calculated from a model forecast (superscript $(\cdot)^f$) of the state vector at time t_i . However, the problem might be high-dimensional so that the solution of (2) becomes computationally intractable especially regarding the real-time requirements of the considered applications. Use of reduced order models [9] might be a possible remedy in this context but is not investigated in this paper.

It is assumed that compared to the true process state, a Gaussian and unbiased model error with known error covariance is made, introducing uncertainty into the calculations.

2.2 Observation Model and Data Assimilation

To alleviate the effects stemming from model uncertainty, the process is also measured by a network of sensors. At every time step t_i , all sensors take a mea-

surement. The resulting observation vector $\boldsymbol{\psi}_i^o$ can be described by the relation

$$\boldsymbol{\psi}_i^o = H_i[\boldsymbol{\chi}_i^t] + \boldsymbol{\epsilon}_i, \quad (3)$$

with the true state $\boldsymbol{\chi}_i^t$ and the unbiased and Gaussian observation error $\boldsymbol{\epsilon}_i$. The observation operator H_i maps vectors from the state space onto the observation space and depends on the measurement positions of the sensors.

To combine results obtained from simulation and from observations, a data assimilation method has to be applied. Most of these methods rely on the formulation that the updated or analysis state vector $\boldsymbol{\chi}_i^a$ results from a linear combination of the forecasted state vector and a weighted innovation due to the observation:

$$\boldsymbol{\chi}_i^a = \boldsymbol{\chi}_i^f + \mathbf{K}_i(\boldsymbol{\psi}_i^o - H_i[\boldsymbol{\chi}_i^f]). \quad (4)$$

The weight matrix \mathbf{K}_i can depend on the model and the observation error covariance matrix as well as on the current estimate's error covariance \mathbf{P}_i^f which is often calculated alongside with the mean estimate. As the estimate's error covariance matrix describes the quality of the state vector, it is especially important for adaptive observations. In this work, the Ensemble Transform Kalman Filter (ETKF) [2] is chosen as data assimilation scheme as it is especially suitable for high-dimensional problems. Furthermore, it is able to calculate the analysis error covariance matrix before the actual measurements are taken.

3 Cooperative Vehicle Control

The model-predictive control (MPC) approach employed in the proposed adaptive observation strategy simultaneously determines collision-free vehicle trajectories as well as optimal target allocation respecting the vehicles' physical characteristics. It can be adapted to various multi-vehicle constellations and task scenarios. Here, it is applied to guide a homogeneous team of sensor platforms to a number of specified target locations while minimizing the distances to attraction points in each vehicle's local environment. Attraction points are updated in every time step, whereas targets are recalculated at greater time intervals, but may shift slowly in the meantime.

Core of the control approach is a discrete-time mixed-integer linear program (MILP) formulation comprising the vehicles' motion dynamics, distance constraints, and several logical expressions. For details on the modeling, the reader is referred to [13]. An optimal control problem of the following form is set up and is solved in a receding horizon fashion to compute optimal control inputs for each vehicle:

$$\min_{U_N} |\mathbf{F}\mathbf{x}^N| + \sum_{k=0}^{N-1} (|\mathbf{G}_1\mathbf{u}^k| + |\mathbf{G}_2\boldsymbol{\delta}^k| + |\mathbf{G}_3\mathbf{z}^k| + |\mathbf{G}_4\mathbf{x}^k|) \quad (5a)$$

$$\text{s.t.} \quad \mathbf{x}^{k+1} = \mathbf{A}\mathbf{x}^k + \mathbf{B}_1\mathbf{u}^k + \mathbf{B}_2\boldsymbol{\delta}^k + \mathbf{B}_3\mathbf{z}^k \quad (5b)$$

$$\mathbf{E}_2\boldsymbol{\delta}^k + \mathbf{E}_3\mathbf{z}^k \leq \mathbf{E}_1\mathbf{u}^k + \mathbf{E}_4\mathbf{x}^k + \mathbf{E}_5. \quad (5c)$$

In this problem formulation, $\mathbf{x} = [\mathbf{x}_c \ \mathbf{x}_b]^T$, $\mathbf{x}_c \in \mathbb{R}^{n_c}$, $\mathbf{x}_b \in \{0, 1\}^{m_b}$, is the system state comprising the vehicle states, the target locations, and each vehicle's local attraction point. $\mathbf{u} = [\mathbf{u}_c \ \mathbf{u}_b]^T$, $\mathbf{u}_c \in \mathbb{R}^{m_c}$, $\mathbf{u}_b \in \{0, 1\}^{m_b}$, comprises the vehicle control inputs. $\boldsymbol{\delta} \in \{0, 1\}^{r_b}$ and $\mathbf{z} \in \mathbb{R}^{r_c}$ represent auxiliary binary and continuous vectors, respectively, e.g. containing distances. The prediction time step $k = 0, \dots, N-1$ relates to the global equidistant time steps $t_i \in \mathbb{N}$ according to $\mathbf{x}^k = \mathbf{x}(t_{i+k})$. As solution of problem (5), the sequence $U_N := \{\mathbf{u}^k\}_{k=0}^{N-1}$ of control inputs is obtained. The first element of U_N is applied to the real system, then its new state is measured for computing updated control inputs at the next time step t_{i+1} . In this manner, the prediction horizon N is shifted over time.

The constraints (5b)–(5c) form a so called mixed logical dynamical (MLD) system, which was proposed in [1] for modeling and controlling constrained linear systems containing interacting physical laws and logical rules. The objective function (5a) can reflect a prioritization of different problem aspects. For the proposed motion planning approach, these are the minimization of each vehicle's distance to its local attraction point, the minimization of distances to still unprocessed target locations, a reward for visiting target locations as well as the minimization of the required control effort. Problem (5) is a mixed-integer linear constrained finite time optimal control (CFTOC) problem. It can easily be transformed into a MILP at each time step of the MPC procedure. Therefore, a numerically robust, efficient computation of control inputs can be performed.

The described control scheme is applied in a centralized manner for the global system of vehicles and targets. Hence, globally optimal cooperative behavior within the scope of the system model and the chosen prediction horizon N is obtained. However, the efficiency of the centralized MPC approach strongly depends on the size of the system model, i.e. the number of vehicles.

4 Adaptive Sensor Motion Strategy

In contrast to related approaches [4, 6, 11, 14, 16, 17], the proposed adaptive observation strategy, in principle, treats state estimation and vehicle control as two separate problems. However, to obtain a working and closed-loop solution, both parts are coupled by information exchange. In short, target and attraction points are calculated based on the error covariance matrix associated with the current state estimate. They serve as input for the vehicle controller responsible for the sensor trajectories. The measurement data obtained from the sensors is then used to update the state estimate and error covariance, and so on.

In detail, the following steps, which are also schematically depicted in Fig. 1, are repeatedly processed:

Determine target points The error covariance matrix provided by the ETKF is a suitable measure of the quality of a state estimate. Large entries indicate high uncertainties, i.e. high deviations between true state and estimate. The objective is to iteratively reduce the entries in the covariance matrix by taking measurements at positions where the uncertainty is largest and, thus, the most valuable information can be obtained.

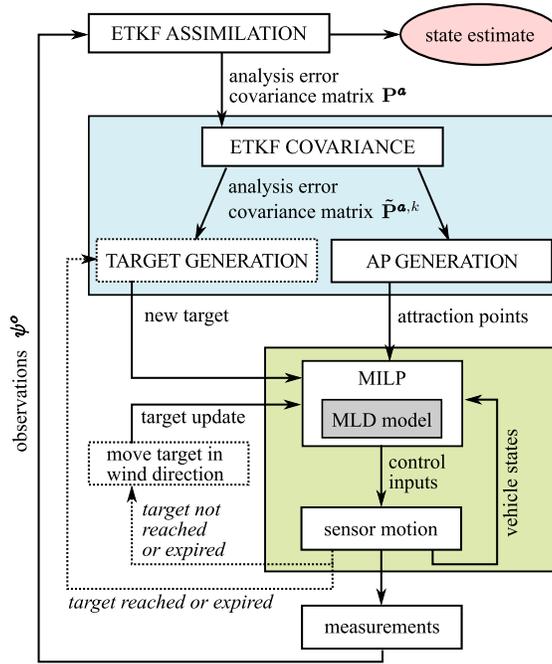


Fig. 1. Overview of the proposed adaptive observation strategy.

In order to determine such a target point, the location corresponding to the maximum value of the diagonal of the current error covariance matrix \mathbf{P}_i^a is chosen. Further target points are calculated iteratively as described above, but considering the analysis error covariance matrix $\tilde{\mathbf{P}}_i^a$. The latter is obtained by applying the ETKF pretending that observations are available at all previously calculated target points, i.e. the observation matrix $\tilde{\mathbf{H}}_i^k$ has to be determined in every iteration k . Hereby, clusterization of target points in regions with high uncertainty is avoided. The procedure is repeated until the number of target points corresponds to the number of sensor vehicles.

The target points then serve as input for the model-predictive controller guiding the sensor vehicles to obtain the corresponding measurement. A measurement at the target point can be considered globally optimum in terms of information gain at that very moment in time.

Determine local attraction points As the sensors are assumed to measure at every time step, the vehicle trajectories leading to the global measurement targets are of importance and should maximize the information gain on the way. For this purpose, local attraction points are introduced that intend to deviate the trajectories into regions of high uncertainty without changing their general orientation towards the target.

Attraction points are determined for each sensor vehicle individually. The calculation equals that of the global targets, but is restricted to the sensor's local environment projected to the next time step assuming the vehicle keeps its current velocity. The circular environment is further reduced to its front half as shown in Fig. 2 in order to preserve the forward motion. In every time step, the attraction points are updated based on the current error covariance matrix.

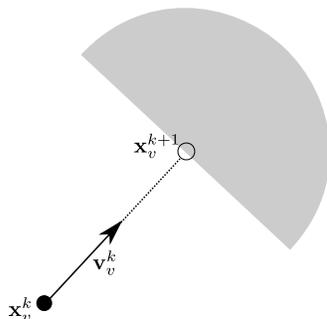


Fig. 2. Local region (gray) for the selection of attraction points based on a projection of the vehicle's position \mathbf{x}_v^k along its current velocity vector \mathbf{v}_v^k .

Control vehicles and process measurements The model-predictive controller provides control inputs for the vehicles, such that each vehicle approaches one of the global targets taking slight detours to stay close to their individual attraction points. In every time step, the obtained measurement data is assimilated to improve the process state estimate and update the error covariance matrix.

Update target points When dealing with dynamic processes, target points might have already lost their optimality by the time their calculation is completed. The information gain of a measurement there more and more decreases over time. That is why different expiration criteria for target points were implemented. Obviously, a target point is replaced as soon as a sensor vehicle has been on the spot to take a measurement. Moreover, a target is discarded if it has not been reached within a certain number of time steps or if the associated uncertainty value is lower than 20% of the current global uncertainty maximum. In all three cases, a new target is generated applying the ETKF-based approach described above. If a target point does not have to be recalculated because of the above criteria, it is moved with the background wind velocity to account for the advective nature of the dynamic process.

5 Results

5.1 Influence of Local Attraction Points

A simplified static test scenario is considered to illustrate the influence of local attraction points on the vehicle trajectories. Two sensor vehicles modeled as point masses with a maximum velocity of 0.015 and a maximum acceleration of 0.01 located on a two-dimensional domain at $(-0.2, -0.4)$ and $(0.5, 0.2)$ are supposed to reach two target points at $(-0.25, 0.25)$ and $(0.5, -0.5)$. To avoid collisions and redundant measurements, the minimum distance between two sensors is set to 0.1. An MPC prediction horizon of $N = 20$ time steps is used, while $\Delta t = 2$. The problem is solved twice - with and without the use of local attraction points. In the scenario with attraction points, those are determined based on a steady background function representing the error variance of the dynamic case.

The results obtained with the two approaches are depicted in Fig. 3. At first, the sensor vehicles mainly try to minimize the distance to both target points until they are close enough to head for the target points. The local attraction points influence the trajectory as the sensors are pulled towards locations with higher values of the background function. If the background function is supposed to represent locations with high uncertainty, the use of the attracted trajectories should produce a higher information gain.

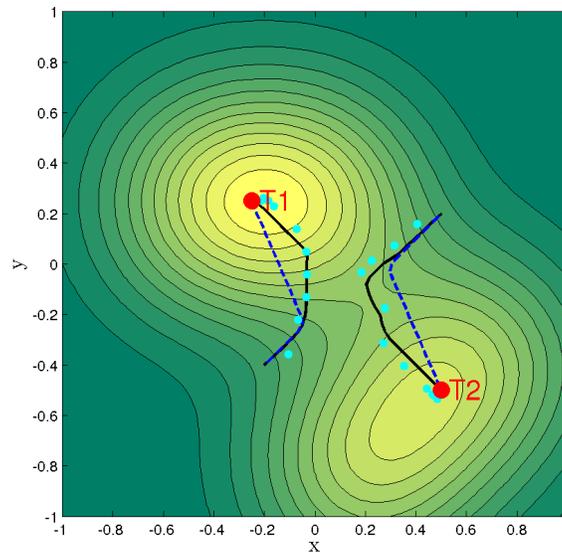


Fig. 3. Unattracted trajectory (blue dashed line) vs. trajectory (black solid line) influenced by attractor points (light blue, every third point is shown).

5.2 Dynamic Test Case

The proposed observation strategy is now applied for state estimation of a two-dimensional dispersion process governed by the source-free linear advection-diffusion equation

$$\frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{w}) - \nabla \cdot (\mathbf{D}\nabla c) = 0, \quad (6)$$

where c represents the concentration to be estimated, \mathbf{D} is the diffusion matrix, which is assumed to be homogeneous and constant, and the velocity field \mathbf{w} is uniform with $w_1 = 0.005$ in x -direction and a vanishing component in y -direction. A finite element method with a characteristic Galerkin approach is applied to discretize and solve (6). The number of grid nodes amounts to about 1000. Note that although the considered problem is kept simple for reasons of clarity, the proposed approach can easily be extended to more sophisticated scenarios, e.g. involving complex velocity fields including eddies.

Observations are obtained by the use of a so-called twin experiment, i.e. the true solution is assumed to be known and it is simulated along with the estimated solution. With this approach, observations can be easily obtained using (3), where the measurement errors are assumed to be uncorrelated and to have a constant variance of 0.01. The difference between the true solution and the estimated solution resides amongst others in their initial condition. A combination of four Gaussian pulses is considered as initial condition. However, for the true initial state and the ensemble generation, the parameters (width, height and position of the pulses) are perturbed by adding numbers drawn from a Gaussian distribution. In total, the ensemble consists of 40 state vectors and localization is used to avoid spurious oscillations [10].

Again, two sensor vehicles with the same configuration as in the static scenario, now starting at (0,0.5) and (0,-0.5), respectively, are considered. For comparison, first, the adaptive observation approach presented in [13] is applied, but slightly modified assuming that the sensors measure at every time step. Then the problem is tackled applying the new motion planning approach involving local attraction points and improved target updates. After 60 time steps, the results depicted in Fig. 4 are obtained.

While using the first approach the vehicles are heading to the middle of the domain to minimize the distance to both target points, the vehicles are attracted more to the boundaries using the proposed extended strategy. The reason for this behavior is a higher uncertainty in the top and bottom area as the uncertainty distribution initially is horseshoe-shaped. Instead of driving through regions where only marginal information can be found, the trajectory is planned to also reduce a high amount of local uncertainty on the way to the target point. This leads to better quality estimates in shorter time.

Having a look on the error confirms this impression. As the true solution is assumed to be known, the quality of the resulting state estimates can be quantified considering their deviation from the true state. Thus, the error can be calculated by forming the norm of the deviation vector. The mean error over time is depicted in Fig. 5(a). Applying the new method with attraction points

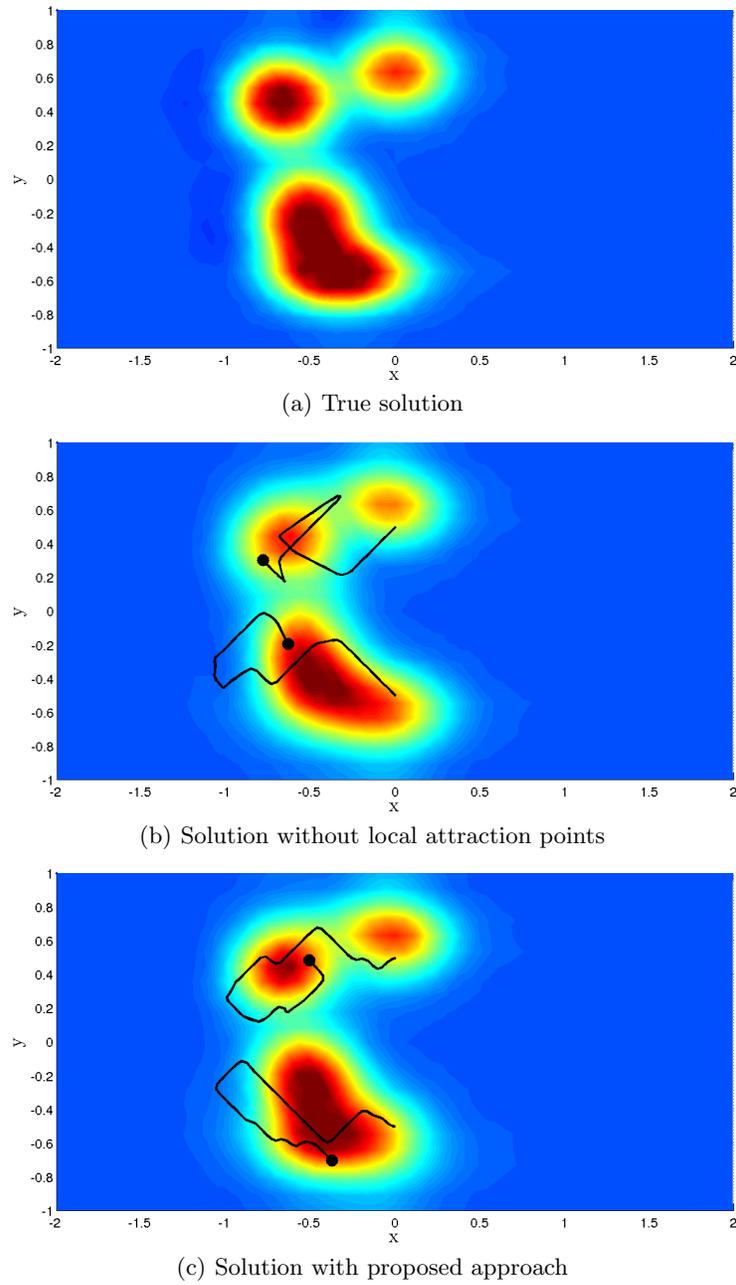


Fig. 4. Concentration distribution at $t = 120$ and related sensor vehicle trajectories

reduces the error much faster than without attraction points. While the error obtained from the simple strategy is around 0.05 at the final time $t = 120$, the proposed strategy in the same time reduces the error to 0.032. The same applies for the norm of the diagonal of the error covariance matrix, which is for the proposed strategy one third less than for the compared strategy (cf. Fig. 5(b)).

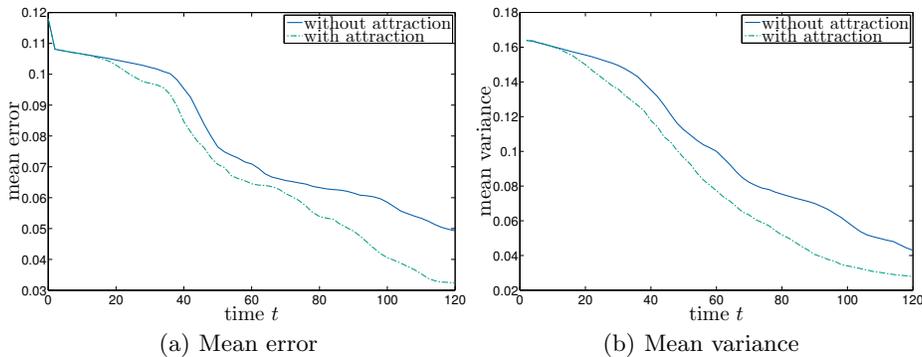


Fig. 5. Mean error and variance over time.

6 Conclusion

A new centralized adaptive observation strategy was presented that relies on a PDE process model, adaptive ETKF-based generation of observation points, and the cooperation of mobile sensors controlled by a MILP-based model-predictive controller. The strategy is especially designed for sensor systems able to measure at high repetition rates as it aims to maximize the uncertainty reduction along the sensor trajectories. Furthermore, a flexible recalculation of target points accounts for the fast evolution of the error covariance. Compared to sensor motion planning without these new features, the proposed method provides estimates of increased quality in shorter time.

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