# Mechanical Influences on the Design of Actuators with Variable Stiffness

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#### 1 Introduction

In the 1990s compliant joint actuation was introduced to robotics with the Series Elastic Actuator [1] and the Mechanical Impedance Adjuster [2]. Such concepts provide safer human-robot interaction, can store energy and decrease force control effort [1]. In the following decades, energy efficient actuation with such concepts had high impact in mobile applications such as bipedal robots, since the stiffness adjustment can be utilized to match the natural frequency of the drive train to the frequency of the desired trajectory [1, 3]. A review on designs of actuators with variable elasticity between drive and link is given in [4] and followed by an energetic examination of specific concepts in [3].

Early solutions for actuators with variable stiffness are designed focusing on the drive side and hence only the input inertia of the drive train is considered as in [1]. Although the input and output inertia of the drive train were included in models of the complete robotic application according to [5], the dynamic interaction of those inertias is not considered for drive train design sufficiently [2]. In contrast to this, input inertia is not considered appropriatly in the dimensioning of recent approaches due to the assumption that drives represent ideal torques or position generators [6, 7, 8, 9]. Further, the analysis of power consumption in [3] is based on the same assumptions and hence does not regard the inertia interaction during dynamic operation. So far, the influence of more than one inertia is mainly considered in complex antagonistically-controlled concepts as AMASC [10] and VSA [11]. While the influence of interaction does not show significant influence on the characteristics of AMASC, its impact is not examined in the case of VSA. In the structurecontrolled and comparable complex VSJ [12], the interaction influence is rather low as in AMASC.

Yet, this is not the case in general applications and thus both inertias and their interaction should be considered, as this has strong impact on the drive train dynamics and hence its dimensioning as well as an energy-efficient strategy for stiffness variation. This paper investigates those influences on a linearized model of a generic actuator with variable stiffness based on the parameters of VTS [13].

### 2 Modelling

To investigate the mechanical influences of input inertia, output inertia and their interaction, simple dynamic models of serial elastic drives are used. Their basic structures are given in Figure 1. While the upper model considers the out-



Figure 1: Investigated drive train models

put inertia  $I_o$  only, the one depicted below can be used to examine the interaction of the inertias. Both are investigated with and without the torque  $m_o g l \sin(\varphi_o)$  at the link due to gravity. The dynamics of those can be represented by their equations of motion: Considering the output inertia only, this is given by the one from [13], while the extended model is given by the equations from [5]. After linearization at equilibrium position, these are transformed to the frequency domain. Thus, the first model with gravity is described by

$$\frac{\varphi_o(s)}{\tau_i(s)} = \frac{1}{I_o s^2 + k + m_o g l},\tag{1}$$

while the transfer function of the extended model is

$$\frac{\varphi_o(s)}{\tau_i(s)} = \frac{k}{I_o I_i s^4 + ((I_i + I_o) k + I_i m_o g l) s^2 + k m_o g l}.$$
 (2)

In both models,  $\varphi_i$  and  $\varphi_o$  represent the input and output position, while  $\tau_i$  and  $\tau_o$  are the corresponding torques. The stiffness *k* is treated as a variable parameter to examine the impact of stiffness variation on the mechanical influences.

## **3** Mechanical Influences

The system dynamics of all four systems with/without  $I_i$  and with/without g - are given in the bode plots shown in Figure 2. Compared to the simple model plotted in blue, significantly higher natural frequencies can be observed for all other models. For the green plot, this is due to an increased stiffness caused by the gravity term, which also leads to an increased static gain. By considering  $I_i$ , the integral behaviour of a rigid body mode can be observed in addition to a higher increase of the natural frequency in the red plot. For the cyan plot, this



Figure 2: Mechanical influences on drive train dynamics

rigid body mode is transformed to an elastic mode with higher frequency due to the fixation introduced by gravity and the elastic mode increases in frequency. In Figure 3, the



Figure 3: Impact of stiffness variation

impact of stiffness variation is presented for the extended model. With increasing stiffness, both natural frequencies increase, where the influence is stronger for the second one.

## 4 Discussion and Conclusion

As the natural frequencies of actuators with variable serial stiffness should be optimized to the specific task for energy-efficient operation, an holistic modelling of the basic mechanical system is required for design. Thus, the mechanical influences of input inertia and gravity should not be neglected in general applications, since the reflected inertia on the input can be comparable to output inertia due to the gear ratio and gravity introduces a virtual fixation. Although drive and link are not rigidly coupled, the reflected inertia is fully present at the input and influences the system dynamics significantly. By considering those mechanical effects, an investigation of power consumption should show two areas with minimum power due to the two natural frequencies. This also has impact on the stiffness variation strategy, since other stiffness values have to be chosen to match the frequencies of actuator and trajectory.

# 5 Format

An oral presentation is preferred by the authors.

# 6 Open Questions

Future work should focus on the investigation of the influence of damping as well as analyses of the eigenvectors, which are definining the relative motion, and the power consumtion of the system.

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