MILP-basierte Optimalsteuerung kooperativer Multi-Vehikel-Systeme

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Die Frage nach der optimalen kooperativen Durchführung von Teamaufgaben mit mobilen Mehrroboter- bzw. Mehrfahrzeug führt im Allgemeinen auf ein nichtlineares diskret-kontinuierliches Optimalsteuerungsproblem. Für Aufgaben, bei denen die Bewegungsdynamik der einzelnen Fahrzeuge entscheidenden Einfluss auf die Qualität der Ausführung haben, besteht häufig eine sehr enge Kopplung von diskreter Entscheidung (z.B. Zuweisung von Rollen oder Teilaufgaben) und kontinuierlicher Trajektorienoptimierung. Zahlreiche praktische Ansätze versuchen eine völlige Entkopplung beider Aspekte, was zwangsläufig einen signifikanten Optimalitätsverlust nach sich zieht. Ziel der vorgestellten Arbeiten ist es, unter Betrachtung des Gesamtproblems methodische Entwicklungen hin zu einer onlinefähigen Regelung zu untersuchen.

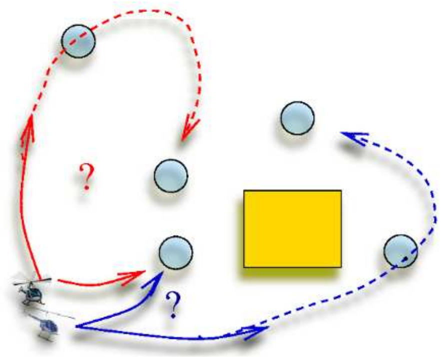

Abbildung 1: Datensammlung an festen Wegpunkten mit Hilfe kooperierender Luftfahrzeuge

Als Ausgangspunkt wird ein nichtlineares hybrides Optimalsteuerungsproblem formuliert, welches sowohl die aufgabenabhängigen Bewegungsdynamiken der Fahrzeuge, logische Bedingungen der Aufgabenverteilung und spezifische Bedingungen der Mehrfahrzeug-Kooperation (z.B. Kollisions- und Formationsbeschränkungen) enthält (vgl. [2]).
Durch Modellvereinfachungen, Linearisierungen und Zeitdiskretisierung wird das Gesamtproblem durch ein gemischt-ganzzahliges lineares Optimierungsproblem (MILP) approximiert, zu dessen Lösung leistungsfähige Methoden bereit stehen. Auf Grund ihrer Effizienz, Robustheit und der geringen Abhängigkeit von Startschätzungen stellt die gemischt-ganzzahlige lineare Optimierung eine sehr attraktive Basis zur Entscheidungsfindung in kooperativen Systemen dar [3].
Lineare hybride Systeme wurden in den vergangenen Jahren intensiv untersucht und dabei insbesondere auch die Frage der MPC-Regelung betrachtet. Die lineare Problemstruktur erlaubt dabei eine multi-parametrische offline-Berechnung des MPC-Gesetzes [1]. Die Anwendbarkeit dieser Ansätze ist Gegenstand aktueller Untersuchungen.

[1] M. Kvasnica. *Efficient software tools for control and analysis of hybrid systems.* PhD thesis, Eidgenössische Technische Hochschule ETH Zürich, 2008.

[2] C. Reinl, M. Glocker, and O. von Stryk. Optimalsteuerung kooperierender Mehrfahrzeugsysteme. *at-Automatisierungstechnik*, 57(6):296–305, 2009.

[3] C. Reinl and O. von Stryk. Optimal control of multi-vehicle systems under communication constraints using mixed-integer linear programming. In *Proc. 1st International Conference on Robot Communication and Coordination (RoboComm)*, Athens, Greece, Oct. 15-17 2007. ICTS.

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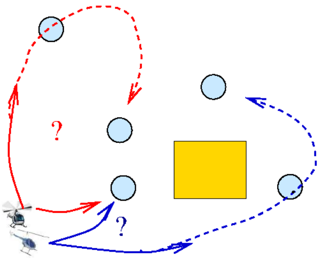
GMA-Fachausschuss 1.40

„Theoretische Verfahren der Regelungstechnik“

Salzburg, 22.09.2009

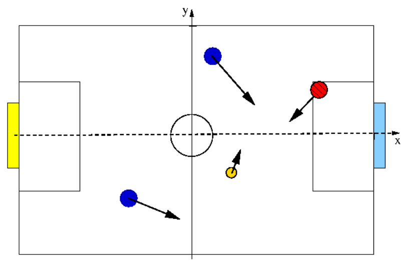


minimize

$$\sum_k \psi_k \cdot (\mathbf{x}(k), \mathbf{u}(k))^T$$

subject to $(\forall\ v_i, j, l, k:)$

$$\mathbf{x}_{v_i}(k+1) - \mathbf{x}_{v_i}(k) = T_s(A_q^{v_i}\mathbf{x}_{v_i}(k) + B_q^{v_i}\mathbf{u}_{v_i}(k))$$
$$\mathbf{C}_{g,q}^{v_i} \leq G_q^{v_i} \cdot (\mathbf{x}_{v_i}(k), \mathbf{u}_{v_i}(k))^T$$
$$a_{v_i,v_j}\, m < C_{v_j} - \tilde{\mathbf{c}}_{v_i} \cdot (x_{v_i}(k), x_{v_j}(k))^T$$
$$(1 - a_{v_i,v_j})M \geq C_{v_j} - \tilde{\mathbf{c}}_{v_i} \cdot (x_{v_i}(k), x_{v_j}(k))^T$$
$$b_{m,coll}(k)\, \mathbf{C}_{m,coll}^{v_i} \leq G_{m,coll}^{v_i} \cdot \mathbf{x}_{v_j}(k)$$
$$b_{j,env}(k)\, \mathbf{C}_{j,env}^{v_i} \leq G_{j,env}^{v_i} \cdot \mathbf{x}_{v_i}(k)$$
$$1 = \sum_{k=1}^{n_s} \sum_{i=1}^{n_v} b_{v_i,l}(k)$$
$$\mathbf{c}_{dis} \leq \mathcal{L}_{dis} \cdot (\mathbf{b}_{con}, \mathbf{b}_{coll}, \mathbf{b}_{env})^T$$



Examples of cooperative multi-robots / multi-vehicle systems

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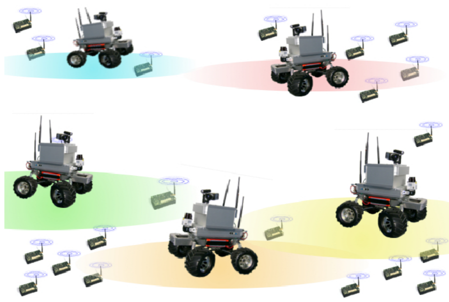
cooperative fire surveillance

source: www.aware-project.net

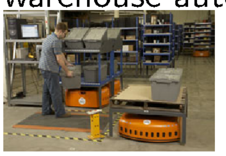
cooperative robot games

source: www.robocup.org

mobile assistance in wireless sensor networks



warehouse automation



www.kivasystems.com

Cooperating mobile robots / vehicles:
discrete decisions and trajectory planning

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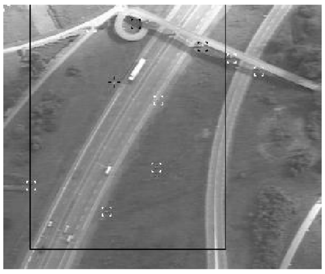
Robot team competition:
Tight coupling of

- robots' motion dynamics and
- role distribution,
- task assignment.

source: www.robocup.org



source: www.comets-uavs.org

Monitoring with unmanned aerial vehicles:
Tight coupling of

- vehicle specific dynamics and
- target assignment,
- waypoint sequencing.

Current research topics

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- appropriate modeling of the multi-vehicle system
- model reduction and abstraction
- system analysis and optimization
- planing and control methods for cooperative multi-vehicle-systems
- application to heterogeneous hardware and various scenarios

⇒ optimal control of cooperation

⇒ consistent bridging of the gap between "exact" methods and heuristic approaches

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Modeling Cooperative Multi-Vehicle Systems

MILP-based Optimal Control of Cooperating Multi-Vehicle Systems

Towards On-line MPC

Summary and Outlook

Problem formulation: some characterizing details

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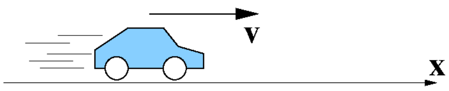
single vehicle i:

- locomotion
  $$\boxed{\dot{\mathbf{x}}^i = \mathbf{f}^i_{q_i}(\mathbf{x}^i(t), \mathbf{u}^i(t))}$$
- constraints
  $$\boxed{\mathbf{g}^i_{q_i}(\mathbf{x}^i(t), \mathbf{u}^i(t)) \leq 0}$$
- allowable sequences of tasks/roles:
  $$\boxed{r^i(b^i_{q_i}(t_s - 0), b^i_{q_i}(t_s + 0)) \leq 0}$$
  $b^i_{q_i}(t) \in \{0, 1\}$, $q_i \in \{1, \ldots, L_i\}$
  $t_s \in \{t_k | k = 1, \ldots, n_s\}$: switching time



cooperative system ($i_1, i_2 \in \{1, \ldots, n_v\}$):

- collision avoidance
  $$\boxed{g_{coll}(\mathbf{x}^{i_1}(t), \mathbf{x}^{i_2}(t)) \leq 0}$$
- underlying geometrical structure
  binary variables: $b_q(t) \in \{0, 1\}$
  mixed constraints
  $$\boxed{[b_q(t) = 1 \Rightarrow C_q(\mathbf{x}^{i_1}(t), \mathbf{x}^{i_2}(t)) \leq 0]}$$
- optimal cooperation
  $$\boxed{\Phi(\mathbf{x}(t), \mathbf{u}(t), \mathbf{b}(t)) \rightarrow \min}$$

Hybrid automata

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$$H = (V, E, \mathbb{X}, \mathbb{U}, ini, \mathbf{f}, \mathbf{j}, \mathbf{i}, \mathbf{e})$$

- Basic components [Henzinger, 1996]:
  - (V, E): finite directed multigraph with knots in V (states) and edges in E (switches)
  - $\mathbb{X}$: set of continuous state variables
  - $\mathbb{U}$: set of continuous control variables
  - ini: map which assigns an initial condition to each edge
  - $\mathbf{i}$: assigns feasible region of state variables to each knot (inequality, equality constraints)
  - $\mathbf{f}$: flow equation or state dynamics for each state
  - $\mathbf{j}$: jump conditions at edges
  - $\mathbf{e}$: events at edges that occurring at switching times
- Advantages and extensions:
  - formal semantics
  - Extension by hierarchies $\rightsquigarrow$ abstraction on different levels
  - Extension by concurrency $\rightsquigarrow$ modeling of multiple vehicles

Hybrid Automaton: resulting trajectories

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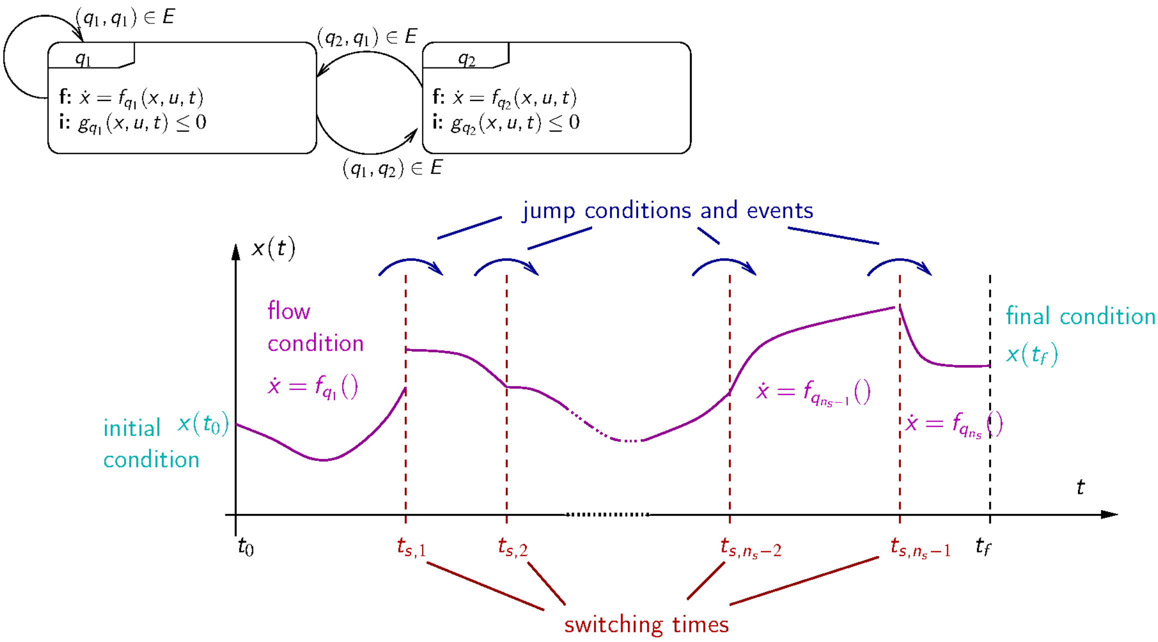

- rules defining feasible sequences of switches

Example 1: robot soccer

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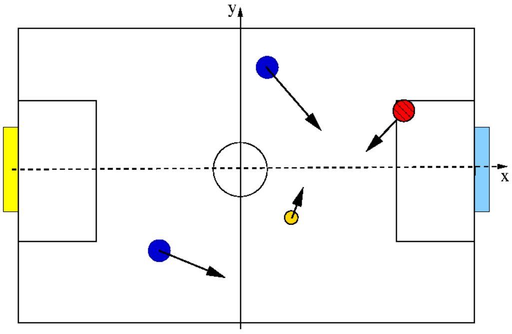

- two controllable attackers (discrete tasks)
- one indirect controllable ball (switched motion dynamics)
- one reactive defender

⇒ dynamically changing environment

Open issues:

- optimize attackers chances for a considered time horizon
- simultaneous task allocation and trajectory planning

⇒ long term goal: model predictive control

Example 1: basic hybrid automaton

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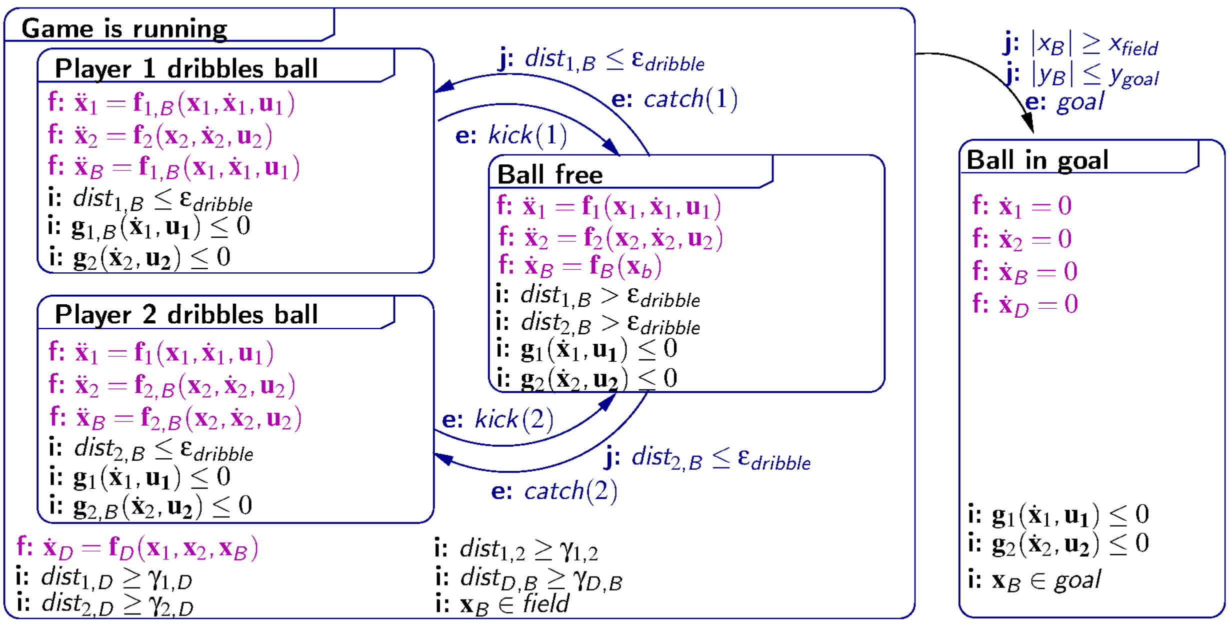

Modeling Cooperative Multi-Vehicle Systems

MILP-based Optimal Control of Cooperating Multi-Vehicle Systems

Towards On-line MPC

Summary and Outlook

Control of multi-vehicle / multi-robot systems

non-linear hybrid optimal control:

[Glocker; Barton & Lee, Rantzer...]

- transformation into finite MINLP
- needs initial guesses and bounds
- maximum principle [Sussmann '99]
- disjunctive programming [Oldenburg & Marquardt '07]

☹ **high-dimensional mixed-integer NLP**

⇳

in practice:

- heuristic approaches to cover all expected situations (e.g. state machines [RoboCup])
- very task specific solutions
- assumptions are just roughly inspired by physics

☹ **no optimality**

⇳

MILP-based optimal control [How '02, D'Andrea '05; Bemporad, Stursberg, Engell ...]

- physics-based approximation
- numerical robustness and efficiency of MILP-solvers
- extension mpMILP for stable MPC
- globally optimal without need of guesses

From HOCP to MILP-based optimal control: linearization and hybridization

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minimize

$$\Phi = \sum_{s=1}^{n_s} (\varphi_s(\mathbf{x}(t_s), \mathbf{u}(t_s), t_s) + \int_{t_{s-1}}^{t_s} L(\mathbf{x}(t), \mathbf{u}(t))\, dt)$$

subject to $(\forall\ i, j, q:)$

$$\begin{aligned}
&\text{(dynamics)} && \dot{\mathbf{x}}^i = \mathbf{f}_q^i(\mathbf{x}^i(t), \mathbf{u}^i(t))\\
& && \mathbf{0} \le \mathbf{g}_q^i(\mathbf{x}^i(t), \mathbf{u}^i(t))\\
& && \mathbf{0} \le \mathbf{h}_q^i(\mathbf{x}^i(t))\\
&\text{(mixed constr.)} && b_{q,i,j}(t) = 1 \Rightarrow C_q(\mathbf{x}^i(t), \mathbf{x}^j(t)) \le 0\\
&\text{(logical constraints)} && \mathbf{L}_q \le \sum_q \sum_{s=1}^{n_s} \sum_{i=1}^{n_v} \pm \mathbf{b}_{i,q}(t_s)
\end{aligned}$$

- polygonal approximation
- hybridization [e.g. Girard '07]
- big-M formulation [Wiliams '96]

minimize

$$\sum_k \psi_k \cdot (\mathbf{x}(k), \mathbf{u}(k))^T$$

subject to $(\forall\ i, j, q, k:)$

$$\begin{aligned}
\mathbf{x}^i(k+1) - \mathbf{x}^i(k) &= \Delta_t (A_q^i \mathbf{x}^i(k) + B_q^i \mathbf{u}^i(k))\\
\mathbf{C}_{g,q}^i &\le \mathbf{G}_{g,q}^i \cdot (\mathbf{x}^i(k), \mathbf{u}^i(k))^T\\
\mathbf{C}_{h,q}^i &\le \mathbf{G}_{h,q}^i \cdot (\mathbf{x}^i(k))^T\\
b_{q,i,j}(k) \cdot M &\ge \tilde{C}_q(\mathbf{x}^i(k), \mathbf{x}^j(k))^T\\
\tilde{\mathbf{L}}_q &\le \sum_q \sum_{k=1}^{n_k} \sum_{i=1}^{n_v} \pm \mathbf{b}_{i,q}(k)
\end{aligned}$$

- linearized difference equation
- fixed sampling time Δ_t

non-linearity $\rightsquigarrow$ more constraints and discrete structure

Objective function $\Phi(\mathbf{x}(t), \mathbf{u}(t), \mathbf{b}(t))$

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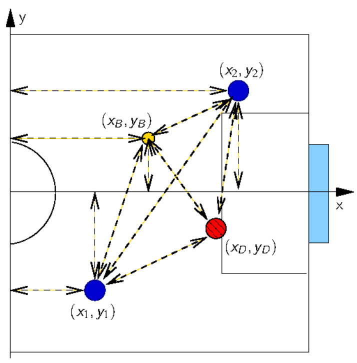
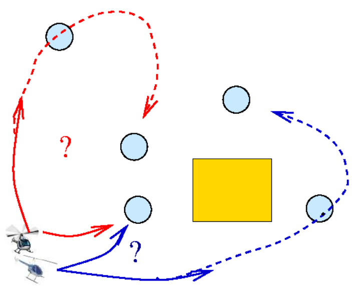
For fixed $[t_0, t_f]$ regarding

- state variables $\mathbf{x}(k)$ (e.g. positions, distances),
- incidence of discrete states $\mathbf{b}(k)$ (e.g. goal, ...),
- control variables $\mathbf{u}(k)$ (e.g. energy input, acceleration).

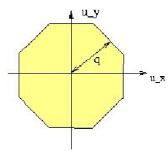




- minimization of energy or "control efforts" for visiting all waypoints

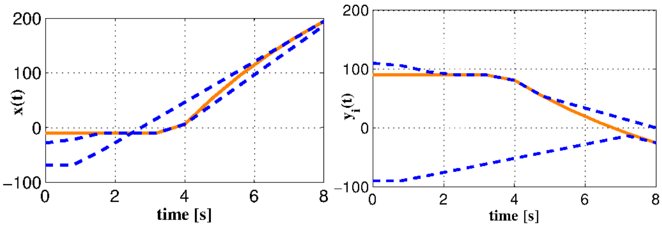
$$\min \int_0^{t_f} u_x(t)^2 + u_y(t)^2\, dt \quad \text{(HOCP)}$$

$$\rightsquigarrow \min \textstyle\sum_k r_k \cdot \Delta_t \quad \text{(lin. approx.)}$$

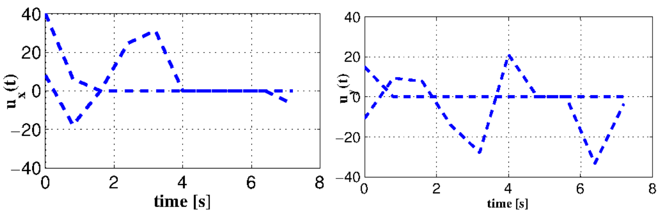


Numerical results: robot soccer

optimal trajectories (x- and y-positions):

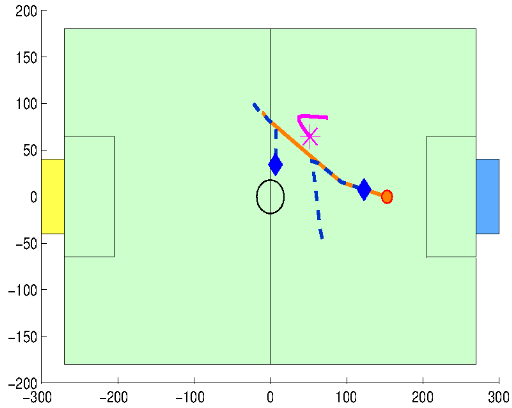


respective optimal control:



- 12 timesteps: 1135 var., 1692 constr.
- linearized dynamics:

$$\boxed{x_{k+1} = x_k + \Delta_t\, v_k\,, \quad v_{k+1} = v_k + \Delta_t\, u_k}$$



- solved with CPLEX in 9 sec.
- computing time strongly depends on initial setting

PC with Intel Pentium M processor (1.86 GHz); 1GB RAM

Example 2: Monitoring with cooperating vehicles

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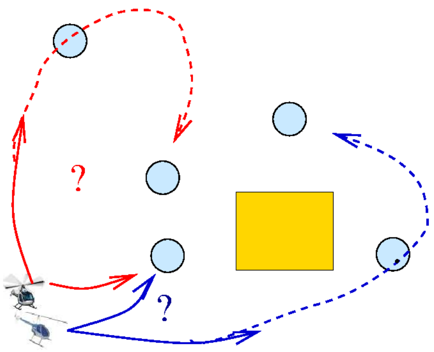
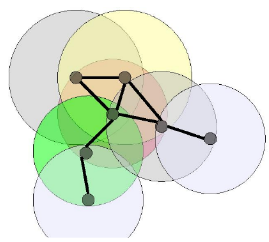


- vehicle-specific motion dynamics (constraints on maximum velocities, controls...)
- certain areas have to be visited during the mission

Extensions:

- structured environment, obstacles
- required connectivity
- dense spatial distribution of many overlapping areas
- ...



$\Rightarrow$ (non-linear) optimal control subject to motion dynamics and heterogeneous, switched constraints

Numerical results: cooperative monitoring

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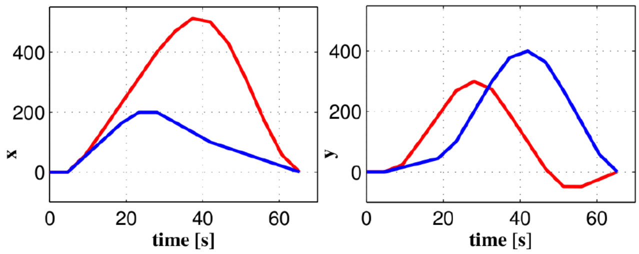
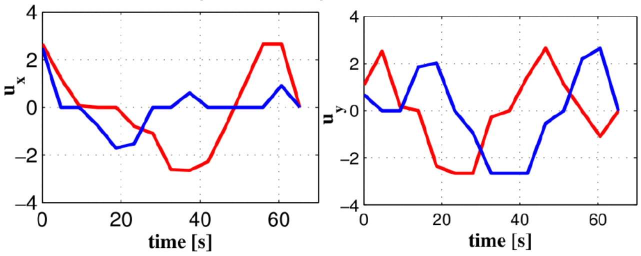
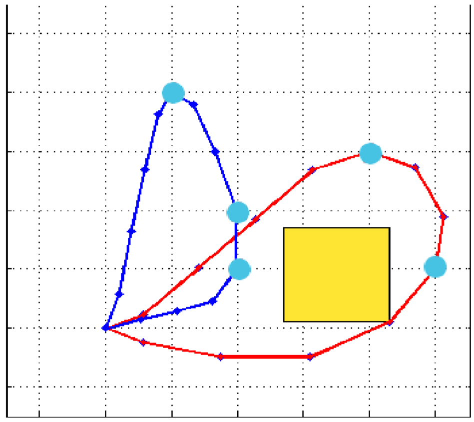
optimal trajectories (x- and y-positions):



respective optimal control:



- simultaneous waypoint sequencing and trajectory optimization



- linearized dynamics:

$$\boxed{x_{k+1} = x_k + \Delta_t\, v_k\,, \quad v_{k+1} = v_k + \Delta_t\, u_k}$$

- solved with CPLEX in 70.2 sec.

PC with Intel Pentium M processor (1.86 GHz); 1GB RAM

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Model predictive control for discrete time linear systems

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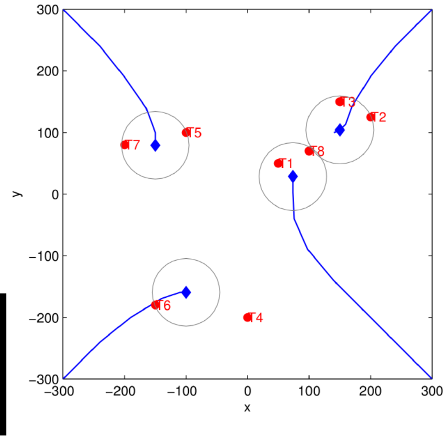
- studied well for MLD and PWA systems [Bemporad, Morari, ...]
- sophisticated implementation: Multi-Parametric Toolbox for Matlab [Kvasnica, Grieder, Baotić; ETHZ]

Example 3: Observing multiple targets with cooperating vehicles:

- multiple cooperating robots
- multiple (moving) targets
- a target is observed, if its position is within the robots observation radius R
- maximize amount and duration of observations

$\Rightarrow$ Discrete-valued optimal control problem subject to motion dynamics and switched constraints



Results: MILP-model - MPC with online optimization

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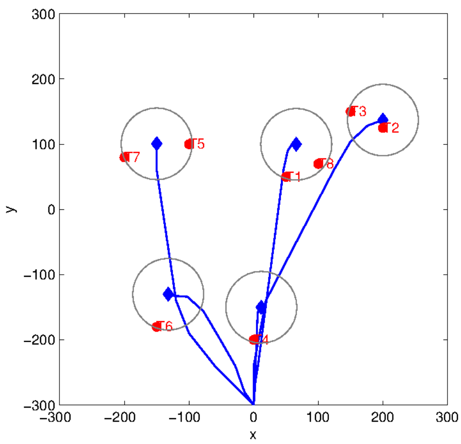
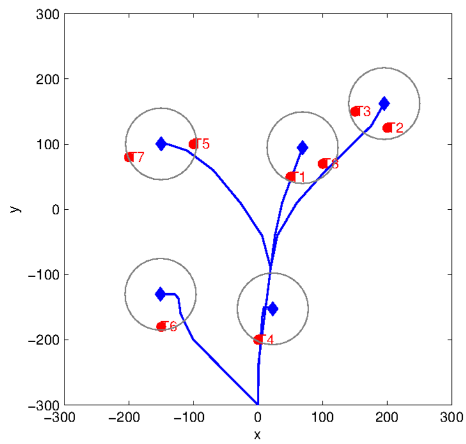
Example: 5 robots, 8 targets, 15 timesteps



MILP

$$\sum_k \sum_r |u_{r,x}^{(k)}| + |u_{r,y}^{(k)}| = 703.46$$



model predictive control

$$\sum_k \sum_r |u_{r,x}^{(k)}| + |u_{r,y}^{(k)}| = 744.02$$

Comparison
MILP-model - Online-MPC-controller

(thanks to J. Kuhn)

Example: 5 robots, 8 targets, 15 timesteps

solving MILP-model:

- full optimal control problem with 15 timesteps
- 10366 constraints
- 1936 variables
- computing time: 2.39h (!) (suboptimal solution after 60 s)

open-loop

MPC with online optimization:[1]

- 15 MPC calls with prediction horizon of 5 timesteps
- 3060 inequality constraints
- 706 Variables
- computing time: $15 \cdot 0.128\,\mathrm{s} = 1.92\,\mathrm{s}$

closed-loop

[1] Matlab MPT-toolbox using CPLEX; running on a PC (Intel Pentium 4 (3.00GHz), 1GB RAM)

Summary and outlook

Summary:

- physics-motivated modeling, analysis and optimization takes account for tight coupling of discrete states and continuous vehicle trajectories,
- MILP enables an efficient simultaneous optimization of cooperation and mobility,
- appropriate for motion dynamics with moderate non-linearity,
- first promising results with MPC-approaches,

Ongoing and related work:

- manage discrete structure in multi-parametric computation,
- extension to more realistic models (e.g. considering uncertainties),
- beneficial combinations with nonlinear approaches and CLP,

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E. Asarin, T. Dang, and A. Girard.
Hybridization methods for the analysis of nonlinear systems.
Acta Informatica, 43(7):451–476, February 2007.

A. Bemporad and M. Morari.
Control of systems integrating logic, dynamics, and constraints.
Automatica, 35(3):407–427, 1999.

M. G. Earl and R. D'Andrea.
Iterative milp methods for vehicle control problems.
In *IEEE Conference on Decision and Control*, volume 4, pages 4369 – 4374, December 2004.

M. Glocker, C. Reinl, and O. von Stryk.
Optimal task allocation and dynamic trajectory planning for multi-vehicle systems using nonlinear hybrid optimal control.
In *Proc. 1st IFAC-Symposium on Multivehicle Systems*, pages 38–43, Salvador, Brazil, October 2-3 2006.

M. Kvasnica.
Efficient software tools for control and analysis of hybrid systems.
PhD thesis, Eidgenössische Technische Hochschule ETH Zürich, 2008.

C. Reinl, M. Glocker, and O. von Stryk.
Optimalsteuerung kooperierender mehrfahrzeugsysteme.
at-Automatisierungstechnik, 57(6):296–305, 2009.

C. Reinl and O. von Stryk.
Optimal control of multi-vehicle systems under communication constraints using mixed-integer linear programming.
In *Proceedings of the. First International Conference on Robot Communication and Coordination (RoboComm)*, Athens, Greece, Oct. 15-17 2007. ICST.