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OPTIMAL TASK ALLOCATION AND DYNAMIC TRAJECTORY PLANNING FOR MULTI-VEHICLE SYSTEMS USING NONLINEAR HYBRID OPTIMAL CONTROL

Markus Glocker, Christian Reinl, Oskar von Stryk

Simulation and Systems Optimization Group, Department of Computer Science, Technische Universität Darmstadt, Hochschulstr. 10, D-64289 Darmstadt, Germany
www.sim.tu-darmstadt.de

Abstract: Based on a nonlinear hybrid dynamical systems model a new planning method for optimal coordination and control of multiple unmanned vehicles is investigated. The time dependent hybrid state of the overall system consists of discrete (roles, actions) and continuous (e.g. position, orientation, velocity) state variables of the vehicles involved. The evolution in time of the system's hybrid state is described by a hybrid state automaton. The presented approach enables a tight and formal coupling of discrete and continuous state dynamics, i.e. of dynamic role and action assignment and sequencing as well as of the physical motion dynamics of a single vehicle modeled by nonlinear differential equations. The planning problem of determining optimal hybrid state trajectories that minimize a cost function as time or energy for optimal multi-vehicle cooperation subject to constraints including the vehicle's motion dynamics is transformed to a mixed-binary dynamic optimization problem being solved numerically. The numerical method consists of an inner iteration where multiphase optimal control problems are solved using a direct collocation method and an outer iteration based on a branch-and-bound search of the discrete solution space. The approach presented in this paper is applied to the scenarios of optimal simultaneous waypoint or target sequencing and dynamic trajectory planning for a team of unmanned aerial vehicles in a plane and to optimal role assignment and physics-based trajectories in robot soccer.

Keywords: multi-vehicle task allocation and trajectory planning, nonlinear hybrid dynamical systems, mixed-integer optimal control, multiple motorized salesmen problem

1. INTRODUCTION

The many different approaches to dynamic coordination of and role assignment for multiple mobile robots range from completely behavior based methods to multi layer architectures. The role assignment or task allocation problem has been addressed, e.g., by the behavior-based architecture *Alliance* Parker (1998) or the publish/subscribe architecture *Murdoch* Gerkey and Matarić (2002) to name only a few. A hybrid systems framework of role assignment in cooperative multi-robot systems has been presented by Chaimowicz et al. (2004). For describing role behavior as well as role and action changes during cooperative task performance a hybrid state automaton is suggested, which consists of a discrete state (role or action) and a continuous state, which is characterized by differential equations of motions and algebraic constraints. The method is investigated for cooperative transport and search tasks. The question of cost functions for optimal role assignment on basis of the hybrid state automaton has not yet been addressed.

For application in the RoboCup Small Size League and in an adversial game called RoboFlag, a method to model complex multi-robot problems has been developed Earl and D'Andrea (2005). Hereby the robots are assumed to have an omnidirectional drive and the behavior of the opponent team is assumed to be known and is described by a discrete state machine. Using a simplified linear dynamics model, the dynamic multi robot planning problems are formulated as mixed logic (linear) dynamic systems Bemporad and Morari (1999). Using a special discretization the problem is approximated by a mixed logic linear time discrete dynamic system and solved using mixed integer linear programming methods.

A standard task for a team of unmanned aerial vehicles (UAVs) in fire monitoring or traffic surveillance is to distribute a certain number of waypoints or targets optimally, so that the individual vehicles' actions optimize an overall objective function. Depending on the scenario, optimality could be measured by the time needed to accomplish a task like in search and rescue applications and/or the energy required for this purpose aiming at reducing the weight of battery packs or at extending a vehicle's operation time. Allocation of targets to UAVs and computing corresponding trajectories are closely connected. Several approaches have been introduced to decompose both into subproblems. For example, estimations of the performance of corresponding trajectories are used to allocate the waypoints Richards et al. (2002) or the tasks are sequentially assigned to the UAVs Furukawa et al. (2004). The combinatorial task assignment problem has been solved using a genetic algorithm Shima et al. (2005) or using conventional multi-TSP problem formulations as studied in the European COMETS project as part of a multi layer decision architecture for a heterogeneous group of UAVs Gancet et al. (2005).

Based on nonlinear hybrid dynamic systems a modeling formalism is developed in this paper which enables a tight and formal coupling of discrete and continuous system state dynamics, i.e. of dynamic role and action assignment and sequencing and the physical locomotion behavior of a cooperating team of mobile robots, and their goal-oriented optimization. Previous approaches for computation of optimal role assignment and robot trajectories in cooperative multi-robot systems need to rely on linear motion dynamic models (e.g. Earl and D'Andrea (2005)) whereas the approach presented in this paper enables to consider nonlinear, physics-based motion dynamics. Compared to previously used models of multi-robot cooperation with general, hybrid dynamic state automata (e.g. Chaimowicz et al. (2004)) we consider the not yet discussed problem of optimization for these general models. A combination of a B&B algorithm and a direct collocation method for numerical optimal control is used to solve the resulting mixed-binary optimal control problem (MBOCP). Applications to two different multi-vehicle example scenarios are given.

2. HYBRID AUTOMATA AND DYNAMICS

A hybrid automaton Henzinger (2000)

$$H = (V, E, \mathbb{X}, \mathbb{U}, init, inv, flow, jump, event) \quad (1)$$

consists of a finite directed multigraph (V, E) with knots in V (called states) and edges in E (so-called switches), a set of continuous state variables $\mathbb{X} = \{x_1, ..., x_{n_x}\}$, a set of continuous control variables $\mathbb{U} = \{u_1, ..., u_{n_u}\}$, a map $init$ which assigns an initial condition to each edge, the invariants provided by the map inv which assigns each knot with a feasible region for the continuous states using equality and inequality constraints, a map $flow$ which assigns a flow equation or state dynamics to each state, a map $jump$ which assigns jump conditions to edges and a map $event$ which assigns events to edges which occur at switches.

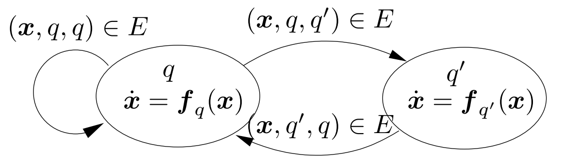


Fig. 1. Basic structure of a hybrid automaton.

A discrete state q combined with the continuous dynamics $\boldsymbol{f}_q = \boldsymbol{f}_q(\boldsymbol{x}, \boldsymbol{u}, t)$ (or $\boldsymbol{f}_q(\boldsymbol{x}, \boldsymbol{u})$ w.l.g.) connected to that state will be referred to as a node of the automaton. The general idea behind is illustrated by Fig. 1. Hybrid automata are well established in the context of robot control (cf. references in Sect. 1).

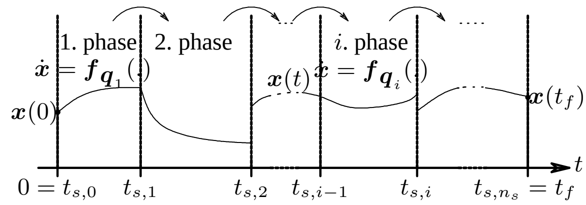


Fig. 2. Illustration of a continuous state trajectory with nonlinear state dynamics defined in n_s phases. Phase transitions occur at switching times $t_{s,i}$.

Starting with some initial condition the system enters the first node of the hybrid automaton and switches at (usually unknown) times $t_{s,i}$ (events) between the nodes. The sequence of nodes and the number of switchings n_s may be given or not. In each node the (piecewise) continuous differentiable state $\boldsymbol{x} : [t_{s,i}, t_{s,i+1}] \rightarrow \mathbb{R}^{n_x}$ (cf. Fig. 2) evolves due to a (piecewise continuous) control variable $\boldsymbol{u} : [t_{s,i}, t_{s,i+1}] \rightarrow \mathbb{R}^{n_u}$ as given by nonlinear flow conditions resulting from the (usually nonlinear) differential equations of motion

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_q(\boldsymbol{x}(t), \boldsymbol{u}(t), t) \,. \quad (2)$$

The discrete state q is constant between two switches and can be represented by an integer valued variable or, equivalently, by a vector of binary variables. Assuming a fixed maximum number of switches for the purpose of optimal planning (Sect. 4), the values of the

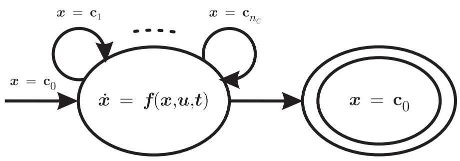


Fig. 3. Hybrid automaton of motorized TSP type.

binary variables over all possible phases can be put together into a binary vector $\boldsymbol{q}_b \in \{0,1\}^{n_{q_b}}$, which represents a sequence of nodes or phases of the hybrid automaton and can also be used to control switches between nodes.

The underlying graph of the hybrid automaton can be expressed by a set of linear constraints, e.g. as

$$0 \geq \boldsymbol{L}\boldsymbol{q}_b, \quad \boldsymbol{L} \in \mathbb{R}^{n_L \times n_{q_b}} . \tag{3}$$

A solution to these constraints represents a feasible sequence of nodes of the hybrid automaton. It should be noted that during optimization (Sect. 4) the linear inequalities can often be solved independently as a feasibility test for the binary control vector $\boldsymbol{q}_b$.

3. EXAMPLE APPLICATIONS

3.1 Application 1: Waypoint sequencing and vehicle trajectories in surveillance

The problem of distributing the selection and sequence of n_c waypoints in a plane (Fig. 6(a)) to be visited by each of the cooperating individual vehicles together with determining optimal vehicle flight trajectories as in a surveillance task represents the more general problem of distributing discrete roles to cooperating vehicles for achieving a common goal. The problem falls in the new class of multiple motorized traveling salesmen problems (TSPs) von Stryk and Glocker (2001). For this considered round-trip problem the automata for one agent looks simple (Fig. 3). After passing the first waypoint (or city) located at $\boldsymbol{c}_i$ the state switches back to the traveling state *go-to-*c_i until the final destination c_0 is reached. Then the system switches into the final state.

For the purpose of demonstration we consider the problem in a horizontal plane as it is usually done in cooperative surveillance planning for multiple UAVs or in air traffic management. The approach presented in this paper also allows a spatial setting. In this case a smooth trajectory must be taken into account and therefore the interconnection between two waypoints is not independent of the rest of the journey as in classical TSPs. In Fig. 4 two different, possible round-trips through four waypoints by one vehicle are depicted.

The vehicle dynamics can be stated as a system of nonlinear differential equations where $\boldsymbol{x}_R$ describes (at least) position, velocity and orientation of the moving vehicle and $\boldsymbol{u}_R$ its control vector. In case of UAVs Eq. (2) may represent as well flight motion in 3-D including aerodynamic effects or, more simplified, 2-D point mass motions in a plane at a certain altitude.

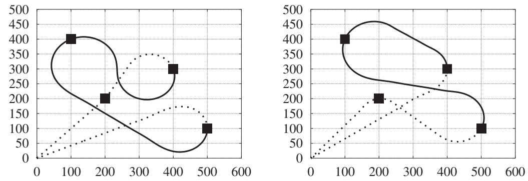


Fig. 4. Two possible round-trips to $n_c = 4$ waypoints.

For the purpose of demonstration we concentrate on the model of a mass point moving in a plane, starting at the origin at initial time $t_0 = 0$ and ending at the origin at the final time t_f. Thus the model reads

$$\begin{aligned} \dot{x}(t) &= v_x(t), \quad x(0) = 0 = x(t_f), \\ \dot{y}(t) &= v_y(t), \quad y(0) = 0 = y(t_f), \\ \dot{v}_x(t) &= a_x(t), \; v_x(0) = 0 = v_x(t_f), \\ \dot{v}_y(t) &= a_y(t), \; v_y(0) = 0 = v_y(t_f), \\ a_x^2 &+ a_y^2 \leq 7 \; . \end{aligned} \tag{4}$$

Hereby $\boldsymbol{x}_R = (x, y, v_x, v_y)^{\mathrm{T}}$ where x, y denote the position, v_x, v_y the corresponding velocities and $\boldsymbol{u} = (a_x, a_y)^{\mathrm{T}}$ the acceleration or braking forces of the vehicle which are constrained. Connecting the possible phases at switching times the boundary condition reads as

$$\bigvee_{j=1}^{n_c} \left[\boldsymbol{x}_R(t_i) - \boldsymbol{c}_j = 0\right] . \tag{5}$$

Using the binary variables $\boldsymbol{q}_{bi}$, $i = 1, \ldots, n_c$, it can be expressed by

$$\begin{pmatrix} x(t_i) \\ y(t_i) \end{pmatrix} - \boldsymbol{C}\boldsymbol{q}_{bi} = 0, \quad i = 1, \ldots, n_c \tag{6}$$

with the matrix $\boldsymbol{C} = (\boldsymbol{c}_1, \ldots, \boldsymbol{c}_{n_c}) \in \mathbb{R}^{2 \times n_c}$ consisting of the waypoint locations, $\boldsymbol{q}_{bi}^{\,T}\boldsymbol{q}_{bj} = 0$ for $i \neq j$ and $\boldsymbol{q}_{bi}^{\,T}\boldsymbol{q}_{bi} = 1$. The system dynamics is not switched in this example but in the following.

3.2 Application 2: Role assignment and physics-based trajectories in robot soccer

The basic actions (i.e. basic behaviors) of an individual soccer playing robot are distinguished, e.g., $q_R \in Q_R = \{$goto ball, dribble ball, kick ball to pos.$\}$ and a role can be composed of several actions. Also for the ball discrete states with different types of motion must be distinguished, e. g., $q_B \in Q_B = \{$free, contact, rebound$\}$.

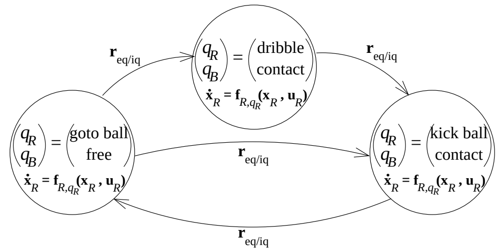


Fig. 5. Example of a hybrid state automaton modeling single robot behavior as a sequence of actions with time dependent discrete and continuous state variables.

The physical motion capabilities of individual mobile, wheeled or legged robots differ significantly. Optimal tactical moves (Fig. 5) can only be achieved if individual locomotion dynamics is considered. This may in general be described in form of Eq. (2) for kinematic vehicle models or kinetic models including the dynamics of the drive train or other desired levels of detail. The control variable $\boldsymbol{u}_R$ represents the control of the kinematic or kinetic robot model used.

For the purpose of demonstration, point mass models of the dynamics of the free moving robot, of the robot dribbling the ball and of the rolling ball are used w.l.g.

$$\begin{aligned}\dot{\boldsymbol{x}}_R &= \overset{\substack{\textbf{Node 1}\\ \text{goto ball}}}{\begin{pmatrix} v_{x,R} \\ v_{y,R} \\ u_{x,R} - F_{x,R} \\ u_{y,R} - F_{y,R} \end{pmatrix}} \vee \overset{\substack{\textbf{Node 2}\\ \text{dribble ball}}}{\begin{pmatrix} v_{x,R} \\ v_{y,R} \\ \alpha_B u_{x,R} - F_{x,R} \\ \alpha_B u_{y,R} - F_{y,R} \end{pmatrix}} \\ \dot{\boldsymbol{x}}_B &= \left(v_{x,B},\, v_{y,B},\, -F_{x,B},\, -F_{y,B}\right)^{\mathrm{T}}\end{aligned} \tag{7}$$

where $\boldsymbol{x}_\diamondsuit = (x_\diamondsuit, y_\diamondsuit, v_{x,\diamondsuit}, v_{y,\diamondsuit})^{\mathrm{T}}$, $\diamondsuit = R, B$, and $F_{*,R} = 2v_{*,R}$, $F_{*,B} = 0.2v_{*,B}$, $* = x, y$, for modeling the force resulting from friction. The control $u_{*,R}$ represents the force to accelerate or decelerate the robot. The coefficient α_B is used as a simple model of the reduced mobility of a player while dribbling the ball. The maximum possible speed of a robot is considered as an inequality constraint

$$0 \le v_{\max}^2 - v_x^2(t) - v_y^2(t) \tag{8}$$

with maximum speed $v_{\max} = 0.32$ cm s^{-1} used in the computations of Sect. 5.2. This can be considered as a general state constraint of type (15). Also several models of the motion behavior of opponents can be incorporated in such a formulation. A continuous state $\boldsymbol{x}_G$ is defined analogously to $\boldsymbol{x}_R$, e.g., $\boldsymbol{x}_G = (x_G, y_G, v_G, \theta_G)^{\mathrm{T}}$. If the position of an opponent (x_G, y_G) is known and constant or if its motion is known as $\boldsymbol{x}_G = \boldsymbol{x}_G(t)$ or $\boldsymbol{x}_G(\boldsymbol{x}_R, \boldsymbol{x}_B, t)$, $0 \le t \le t_f$, it can be considered as a (reactive) moving obstacle in the formulation of the constraints (15) as $\boldsymbol{g}_{R,q_{R,i}}(\boldsymbol{x}_R(t), \boldsymbol{x}_G(t), t) \ge 0$ (e.g. Sect. 5.2, Fig. 8).

The hybrid state of the system consisting of one robot and one ball is described by $(\boldsymbol{q}, \boldsymbol{x})$ where $\boldsymbol{q} = (q_R, q_B)^{\mathrm{T}}$ and $\boldsymbol{x} = (\boldsymbol{x}_R, \boldsymbol{x}_B)^{\mathrm{T}}$. In the same manner the hybrid state of a system with n_R cooperating robots and one ball can be described as

$$\begin{aligned}\boldsymbol{q} &= (q_{R_1}, \dots, q_{R_{n_R}}, q_B)^{\mathrm{T}} \\ \boldsymbol{x} &= (\boldsymbol{x}_{R_1} \dots, \boldsymbol{x}_{R_{n_R}}, \boldsymbol{x}_B)^{\mathrm{T}}\end{aligned} \tag{9}$$

and continuous state dynamics as in Eq. (2) with $\boldsymbol{f} = (\boldsymbol{f}_{R_1} \dots, \boldsymbol{f}_{R_{n_R}}, \boldsymbol{f}_B)^{\mathrm{T}}$, control $\boldsymbol{u} = (\boldsymbol{u}_{R_1} \dots, \boldsymbol{u}_{R_{n_R}})^{\mathrm{T}}$. In addition, the right hand side $\boldsymbol{f}$ usually depends on the discrete state $\boldsymbol{q}$, i.e. $\boldsymbol{f} = \boldsymbol{f}_{\boldsymbol{q}}(\boldsymbol{x}, \boldsymbol{u}, t)$ as in Eq. (7).

Furthermore, the transition from one node (phase) to another requires that jump or switching conditions must be satisfied. For example, the transition from the discrete state $q_R = \text{goto ball}$ to $q_R = \text{dribble ball}$ requires, that the (x, y)-coordinates of the robot and the ball must be equal or within a certain distance. Also the orientation of the robot must allow the robot to bring the ball under control, which may not be possible if the ball reaches the robot in its back.

4. NONLINEAR HYBRID OPTIMAL CONTROL

4.1 Hybrid optimal control problem statement

For a feasible sequence of n_s discrete system states

$$\boldsymbol{q}_i := \boldsymbol{q}(t),\ t_{s,i-1} \le t \le t_{s,i},\ i = 1, \dots, n_s, \tag{10}$$

where $t_{s,0} := 0$ and $t_{s,n_s} := t_f$, the initial state $\boldsymbol{x}(0)$ and also the control history $\boldsymbol{u}(t)$, $0 \le t \le t_f$, are given, the system trajectory $\boldsymbol{x}(t)$, $0 \le t \le t_f$, can under mild assumptions uniquely be determined from

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}_{\boldsymbol{q}_i}(\boldsymbol{x}(t), \boldsymbol{u}(t), t),\ t_{s,i-1} < t < t_{s,i}, \tag{11}$$

$i = 1, \dots, n_s$, considering also the jump or switching conditions (e.g. when a waypoint is reached in the first example or when a robot is close enough to the ball to start dribbling in the second example)

$$\begin{aligned}0 &= \boldsymbol{r}_{\mathrm{eq},\boldsymbol{q}_{i-1},\boldsymbol{q}_i}\left(\boldsymbol{x}(t_{s,i} - 0), \boldsymbol{x}(t_{s,i} + 0)\right) \\ 0 &\le \boldsymbol{r}_{\mathrm{iq},\boldsymbol{q}_{i-1},\boldsymbol{q}_i}\left(\boldsymbol{x}(t_{s,i} - 0), \boldsymbol{x}(t_{s,i} + 0)\right)\end{aligned} \tag{12}$$

$i = 1, \dots, n_s - 1$, where $t \pm 0 := \lim_{\epsilon \to 0, \epsilon > 0} t \pm \epsilon$.

Now we consider the *hybrid optimal control problem* where we wish to determine the optimal sequence of actions of the cooperating robots, i.e. the discrete state values $\boldsymbol{q}_i$, $i = 1, \dots, n_s$, $n_s \le n_{s,\max}$, as well as the continuous control history $\boldsymbol{u}(t)$, $0 \le t \le t_f$, and the switching times $t_{s,1}, \dots, t_{s,n_s} = t_f$ in a way that the cost function

$$\min_{\boldsymbol{u}, \boldsymbol{q}_1, \dots, \boldsymbol{q}_{n_{s,\max}}} J, \tag{13}$$

$$\begin{aligned}J = {} & \varphi_{n_s}(\boldsymbol{x}(t_f), t_f) + \sum_{i=1}^{n_s-1} \varphi_i\left(\boldsymbol{x}(t_{s,i} - 0), \boldsymbol{x}(t_{s,i} + 0)\right) \\ & + \sum_{i=1}^{n_s} \int_{t_{s,i-1}}^{t_{s,i}} L_i\left(\boldsymbol{x}(t), \boldsymbol{u}(t), t\right) \mathrm{d}\,t\end{aligned}$$

with real-valued functions φ_i, L_i is minimized subject to the equations of motion (11), the initial condition $\boldsymbol{x}(0) = \boldsymbol{x}_0$ and the switching conditions (12), constraints on the final state

$$0 = \boldsymbol{r}_{\mathrm{eq},\boldsymbol{q}_{n_s}}\left(\boldsymbol{x}(t_f)\right),\ 0 \le \boldsymbol{r}_{\mathrm{iq},\boldsymbol{q}_{n_s}}\left(\boldsymbol{x}(t_f)\right), \tag{14}$$

constraints on the (continuous) state and control variables in $(t_{s,i-1}, t_{s,i})$, $i = 1, \dots, n_s$,

$$0 \le \boldsymbol{g}_{\boldsymbol{q}_i}(\boldsymbol{x}(t), t), \tag{15}$$

$$\begin{aligned}\boldsymbol{u}_{\boldsymbol{q}_i,\min} &\le \boldsymbol{u}(t) \le \boldsymbol{u}_{\boldsymbol{q}_i,\max}, \\ \boldsymbol{x}_{\boldsymbol{q}_i,\min} &\le \boldsymbol{x}(t) \le \boldsymbol{x}_{\boldsymbol{q}_i,\max},\end{aligned} \tag{16}$$

with constant lower and upper bounds. In general not all actions $\boldsymbol{q}_i \in Q$, where Q only consists of feasible combinations of the discrete states of considered vehicles or robots, can follow or proceed each other. Thus, additional constraints must be considered, e.g. in a form as in Eq. (3).

4.2 *Numerical solution*

If a maximum number of switching times (i.e. discrete state transitions) is assumed, then an unknown sequence of discrete state variables $\boldsymbol{q}_i = \boldsymbol{q}(t) \in Q$, $t_{s,i-1} < t < t_{s,i-1}$, (Eq. (10)) can be transformed to an integer variable $\boldsymbol{i}_\mathrm{q} \in I_\mathrm{q} \subset \mathbb{Z}^{n_i}$ which can be represented by a vector of binary variables $\boldsymbol{q}_b \in \{0,1\}^{n_{q_b}}$. The feasibility of succeeding or preceeding actions, phases or nodes is described by constraints as in Eq. (3). Thus, the previously introduced hybrid optimal control problem is transformed into a mixed-integer, namely mixed-binary, dynamic optimization problems and numerical methods for this class of problems can be applied.

The numerical solution approach consists of a decomposition of MBOCP in coupled discrete and dynamic optimization problems at outer and inner levels (see Buss et al. (2002); von Stryk and Glocker (2001) for details). At the inner optimization level dynamic optimization problems are considered of which the nonlinear state dynamics is defined on multiple phases (Fig. 2). For each phase $[t_{s,i-1}, t_{s,i}]$ a time discretization grid is introduced. Along this time grid the continuous state variables $\boldsymbol{x}(t)$ and the control variables $\boldsymbol{u}(t)$ are approximated by piecewise polynomial functions von Stryk and Glocker (2001). Thus, the dynamic optimization problem is transformed into a large, sparse nonlinear constrained optimization problem which is solved numerically by a sparse sequential quadratic programming method Gill et al. (2002). At the outer iteration level an investigation of the discrete solution space is performed. For this purpose, B&B methods are applied. Their performance depends on maintaining good lower and upper bounds on the cost function (13) (cf. von Stryk and Glocker (2001)).

It should be noted that the briefly outlined numerical solution approach aims at the general case of nonlinear hybrid dynamical systems. In special cases, e.g. for clocked hybrid piecewise affine systems Bemporad and Morari (1999), the optimal control problem can be transformed to a mixed-integer linear program which can be solved more easily by efficient methods from discrete optimization.

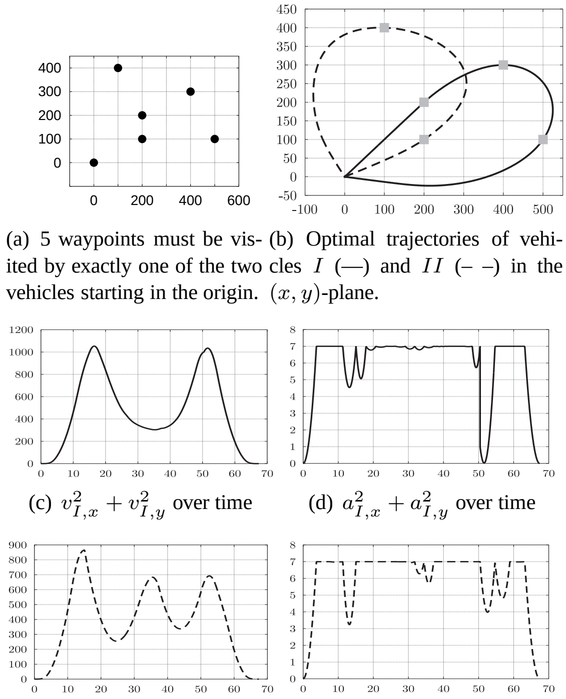
5. RESULTS

5.1 *Optimal vehicle trajectories and sequencing*

First we consider the problem of determining the optimal sequence of four waypoints and physics-based trajectories for the round-trip of a vehicle starting in the origin as described in Sect. 3.1. As cost function (13) for the multiphase MBOCP

$$\min_{\boldsymbol{u},\, q_b} J[\boldsymbol{u}, q_b] := t_f + 0.002 \int_0^{t_f} (a_x^2 + a_y^2)\,\mathrm{dt} \quad (17)$$

has been chosen which must be minimized subject to the vehicle dynamics model (4), initial and final



(a) 5 waypoints must be visited by exactly one of the two vehicles starting in the origin.

(b) Optimal trajectories of vehicles I (—) and II (– –) in the (x, y)-plane.

(c) $v_{I,x}^2 + v_{I,y}^2$ over time

(d) $a_{I,x}^2 + a_{I,y}^2$ over time

(e) $v_{II,x}^2 + v_{II,y}^2$ over time

(f) $a_{II,x}^2 + a_{II,y}^2$ over time

Fig. 6. Solution for two cooperating vehicles

conditions and constraints as well as the switching conditions (6). A computed solution of this problem for an optimal round-trip of one vehicle from the origin to four waypoints including optimal switching, i.e. visiting, times is depicted in Fig. 4 right.

Next, we consider the problem of five waypoints which must be visited by exactly one of two cooperating vehicles I and II starting in the origin (Fig. 6(a)). As cost function the overall time needed to visit the cities and to complete the round-trip may be used. But the solution may not be unique, because it is not defined, what the faster vehicle does, while the slower one is still on its way. Uniqueness of the solution can be achieved, if a sum of time and energy consumption of the vehicles is minimized as in Eq. (17).

For the problem formulation each of the waypoints is associated with a further binary variable $\hat{q}_{b\,i}$ indicating which of the two vehicles will visit it

$$\hat{q}_{b\,i} \begin{pmatrix} x_I(t_i) \\ y_I(t_i) \end{pmatrix} + (1 - \hat{q}_{b\,i}) \begin{pmatrix} x_{II}(t_i) \\ y_{II}(t_i) \end{pmatrix} - \boldsymbol{C}\boldsymbol{q}_{b\,i} = 0,$$

for $i = 1, \ldots, n_c$. As six waypoints are assumed, the problem consists of $2\sum_{k=1}^{5} k! = 306$ possible discrete solutions including those where one of the vehicles visits all waypoints. In Fig. 6(b) a computed solution of the optimal vehicle trajectories and waypoint assignment and sequencing problem is depicted.

5.2 *Optimal role assignment and robot trajectories*

As an example of the application given in Sect. 3.2 we consider the task-oriented dynamic role assignment and trajectory optimization of two strikers. Three different discrete states are considered

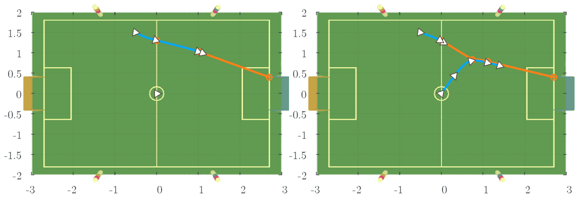


Fig. 7. Optimal trajectories for two strikers with fast (left) and slow (right) dribbling capabilities.

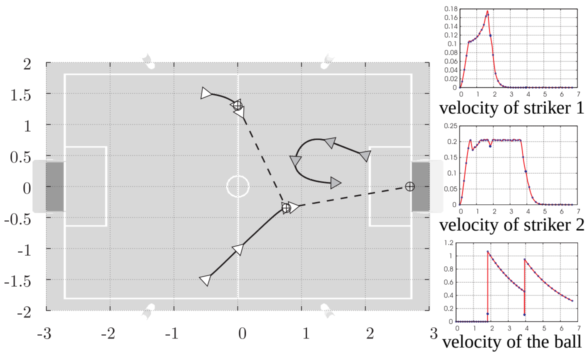


Fig. 8. Solution of the two strikers versus one defender example in state space (x, y).

$$\begin{bmatrix} \textbf{Node 1} \\ \text{“Player one dribbles ball”} \\ \ddot{\boldsymbol{x}}_1 = \boldsymbol{f}_{1,B}(\boldsymbol{x}_1, \dot{\boldsymbol{x}}_1, \boldsymbol{u}_1) \\ \ddot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(\boldsymbol{x}_2, \dot{\boldsymbol{x}}_2, \boldsymbol{u}_2) \\ \boldsymbol{x}_B = \boldsymbol{x}_1 \end{bmatrix} \vee \begin{bmatrix} \textbf{Node 2} \\ \text{“Player two dribbles ball”} \\ \ddot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1, \dot{\boldsymbol{x}}_1, \boldsymbol{u}_1) \\ \ddot{\boldsymbol{x}}_2 = \boldsymbol{f}_{2,B}(\boldsymbol{x}_2, \dot{\boldsymbol{x}}_2, \boldsymbol{u}_2) \\ \boldsymbol{x}_B = \boldsymbol{x}_2 \end{bmatrix}$$
$$\vee \begin{bmatrix} \textbf{Node 3} \\ \text{“Ball free”} \\ \ddot{\boldsymbol{x}}_1 = \boldsymbol{f}_1(\boldsymbol{x}_1, \dot{\boldsymbol{x}}_1, \boldsymbol{u}_1) \\ \ddot{\boldsymbol{x}}_2 = \boldsymbol{f}_2(\boldsymbol{x}_2, \dot{\boldsymbol{x}}_2, \boldsymbol{u}_2) \\ \ddot{\boldsymbol{x}}_B = \boldsymbol{f}_B(\boldsymbol{x}_B, \dot{\boldsymbol{x}}_B) \end{bmatrix}.$$

For the purpose of demonstration, the state kick-ball-to-pos. is assumed to take place instantaneously at a switching time. The task of the two strikers depicted by white triangles in Figs. 7 and 8 is to play the ball, whose initial position is in rest in the upper half of the middle line, into the goal and to minimize a cost function (13) where

$$\begin{aligned} &\varphi_{n_s} = \rho_t \cdot t_f, \;\; \varphi_i = 0, \; i = 1, \ldots, n_s - 1, \\ &L_i = \rho_e \cdot \|\boldsymbol{u}(t)\|_2^2, \; i = 1, \ldots, n_s, \end{aligned} \tag{18}$$

with constant weights ρ_e, ρ_t.

First, the effect of different locomotion properties is studied (Fig. 7). A faster dribbling capability of a robot ($\alpha_B = 0.9$) leads to dribbling and direct kick to the goal by the robot close to the ball. For a slower dribbling capability ($\alpha_B = 0.1$) it is faster to kick the ball into the goal if a double pass is applied.

Next, it is assumed that the there is one defending robot which has a known reactive behavior and moves with its maximum speed from its initial position towards the current position of the ball. The two attacking players must ensure that the ball comes not closer to the defender than a certain distance. Sequence of actions of both strikers, switching times of the phase transitions and robot trajectories are obtained as the numerical solution of a nonlinear hybrid optimal control problem with five phases (Fig. 8).

6. DISCUSSION AND OUTLOOK

The computational time for solving the scenarios on a recent PC ranges from a few minutes for scenario 5.2 to about half an hour for scenario 5.1. The computational time easily scales up enormously by increasing the number of robots or vehicles involved, the number of discrete states (waypoints or basic behaviors) or the dimension and nonlinearity of the vehicle dynamics. But improvements in hybrid optimal control methods can be expected in the next years. The presented approach is especially suited as an offline planning or design method for multi-vehicle problems where utilizing the locomotion dynamics is essential for successful task achievement and where reasonably accurate simulation models are available. The hybrid optimal control solution also allows to evaluate the performance of real-time capable, but heuristic and approximative multi-vehicle control methods. General models of vehicle dynamics can be included in (2) as well as different sensor models of the vehicles as constraints in (15).

As a consequence of the principle of optimality in optimal control all vehicles having exactly the same world model information and solving the optimal control problem individually will obtain the same solution. However, noise and data inconsistencies must be considered for real robots. Synchronization will be required during execution of plans. Communication between vehicles can be included in the formulation with hybrid automata as an additional state. Stability of the trajectories against disturbances can be included to some extent in the hybrid optimal control problem formulation.

Ongoing work considers a systematic modeling of nonlinear and linear, e.g. piecewise affine, hybrid optimal control problems and aims at the development of online planning methods based on mixed-integer linear programming and their combination with hierarchical state machines for behavior programming for individual robots and teams Loetzsch et al. (2006).

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