Introduction to

Numerische Lösung optimaler Steuerungsprobleme: Diskretisierung, Parameteroptimierung und Berechnung der adjungierten Variablen by Oskar von Stryk,

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Many complicated technical processes can mathematically be described as solutions of systems of ordinary differential equations. The investigation of optimal solutions leads to optimal control problems. Examples are the optimal abort landing of a Boeing 727 in the presence of a windshear (Chapter 8.2), the maximum range flight of a hang glider through an upwind (Chapter 8.3), the minimum time and minimum energy trajectories of industrial robots (Chapters 8.4 and 8.5), as well as the design of high frequent oscillators with minimized phase noise (Chapter 8.6).

The dynamics is described by several highly nonlinear equations. Also, several constraints have to be satisfied. Therefore, the solutions of such optimal control problems can only be done efficiently by numerical methods.

This work is concerned with new and efficient methods for solving optimal control problems: By discretizations of state and control variables the infinite dimensional optimal control problems are transformed into optimization problems in finite dimensional spaces.

Two specific discretizations are investigated. Constraint functions and dynamic equations are only satisfied at discrete points (direct collocation). Under certain conditions, a solution of the discretized problem approximates a solution of the continuous problem if the discretization is fine enough.

By this approach, a reliable computation of adjoint variables and of the often complicated switching structures of state and control constraints also becomes possible. The method provides with excellent initial values for a subsequent treatment of the optimal control problem by the (indirect) multiple shooting method [24], [136]. The computational and time consuming efforts of establishing homotopy chains with various switching structures by the multiple shooting method can significantly be reduced. The efficiency of this "hybrid" method, the combination of direct collocation and multi-

ple shooting method, is demonstrated among other problems by the optimal abort landing of a Boeing 727 in the presence of a windshear and by the minimization of time and energy for trajectories of industrial robots.
The optimal robot trajectories computed by the new numerical method are well suited for an implementation as successful experiments prove.

Within this work, the algorithm DIRCOL for the numerical solution of optimal control problems has been developed and implemented. A listing of the Fortran code has been omitted due to its length.

The solution of the resulting finite dimensional nonlinear optimization problems is done by the algorithm NPSOL due to Gill, Murray, Saunders, and Wright [51] that is based on Sequential Quadratic Programming.

DIRCOL has been successfully applied for the numerical solution of new and complicated optimal control problems. The algorithm is currently also successfully used at several Institutes of the Technische Universität München: at the Lehrstuhl für Höhere Mathematik und Numerische Mathematik (Numerical Analysis) in several Diploma theses and research projects (cf. Chapter 8.7), at the Lehrstuhl für Hochfrequenztechnik (High Frequency Techniques) (cf. Chapter 8.6), and at the Lehrstuhl für Elektrische Antriebstechnik (Electrical Driving Techniques) (cf. Chapter 8.4.4).

In **Chapter 1** a description of the mathematical problem is given. A general optimal control problem (P) is introduced. The theoretical investigations are done for problem (P). An extension of (P) to problems with several phases leads to the more general class of problems (AP). The algorithm DIRCOL has been developed for the problem class (AP).

Chapter 2 gives a short review on commonly used numerical methods for optimal control problems.

The discretizations of optimal control problems are described in **Chapter 3**.

Chapter 4 is concerned with convergence investigations. The solution of the necessary first order conditions of the discretized problem does under certain conditions approximate a solution of the necessary conditions of optimality of the continuous problem if the discretization is fine enough.

In **Chapter 5** a method to compute the adjoint variables of the optimal control problem by a solution of the discretized problem is described. This method works also reliable in the presence of active state constraints. By an estimate of the adjoint variables an estimate of the error of the objective functional is possible (Chapter 5.4). This error estimator can be very reliable and may be accurate not only in the magnitude of the error but also up to the first digit. The results for the maximization of the flight range of a hang glider through an upwind (Chapter 8.3) and for the design optimization of high frequent oscillators (Chapter 8.6) show this.

A proper selection of the discretization points is important for the efficiency of the method. Therefore, several new and efficient methods for a static and movable selection of the grid points of the discretization are introduced in **Chapter 6**. The efficiency of the grid point selection is demonstrated in detail by the optimal tracking of a prescribed path for an idealized two-link robot arm (Chapter 6.4).

Further aspects of an efficient implementation as a proper statement of the optimal control problem, the computation of gradients, SQP-methods, stopping criteria, etc. are investigated in **Chapter 7**.

Chapter 8 contains selected applications of the new method.
By the problem of optimal abort landing of a Boeing 727 in the presence of a windshear (Chapter 8.2) it is demonstrated how the derivation and computation of a homotopy chain of solutions using the multiple shooting method requiring several weeks can be reduced to a few hours by using DIRCOL.
The solution as well as the adjoint variables and entry and exit points of the control constraint are accurately computed for the maximum range trajectory of a hang glider through an upwind (Chapter 8.3).
The minimization of point-to-point trajectories with respect to time and energy of the end effector of an industrial robot of type Manutec r3 is investigated in detail in Chapter 8.4. The efficiency of the combination of the direct collocation and the multiple shooting method is demonstrated by the solution of a complicated optimal control problem where several state constraints of different orders are active. For example, a minimum time trajectory with eleven nearby switching points has been computed. Not only the switching structure but also the computed estimates of the adjoint variables are as accurate as the multiple shooting method requires only a few iteration steps to find the solution accurate to ten digits (Chapter 8.4.2). Furthermore, the

switching structure can also be considered in the optimization with DIRCOL itself. Despite of the high complexity of the dynamic model and of the many state and control constraints the solution can also be computed by DIRCOL in high accuracy if the switching structure is included in the discretization (Chapter 8.4.3).
The computed optimal robot trajectories are well suited for an implementation in real robots as experiments prove.
A comparison of the new method with several known methods for the computation of minimum time robot trajectories in two, three and six degrees of freedom shows that the published results are not always optimal solutions (Chapter 8.5).

Finally, the wide applications of the new method are demonstrated by the problem of minimum noise design of high frequent oscillators (Chapter 8.6). The minimum noise design for a specific high frequent oscillator can be computed for the first time by using DIRCOL. The new oscillator has been manufactured and measurements of it prove the significant reduction of the phase noise [4].